LECTURE: 2-6 LIMITS AT INFINITY

Intuitive Definition of a Limit at Infinity Let f be a function defined on some interval (a, ∞) or $(-\infty, a)$. Then

$$\lim_{x\to\infty} f(x) = L \quad (\text{or } \lim_{x\to-\infty} f(x) = L)$$

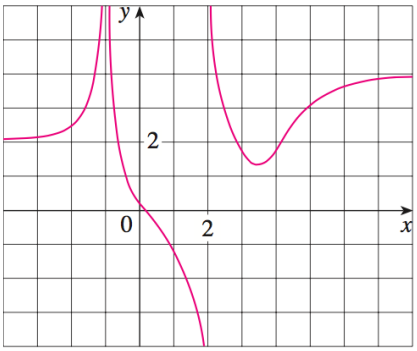
means that the values of $f(x)$ can be made arbitrarily close to L by requiring x to be big enough or

The line $y = L$ is called a **horizontal asymptote** of the curve $y = f(x)$ if either

$$\lim_{x\to\infty} f(x) = L \quad \text{or} \quad \lim_{x\to-\infty} f(x) = L$$

Example 1: Sketch a graph of $y = \tan^{-1} x$ and find the $\lim_{x\to\infty} \tan^{-1} x$ and $\lim_{x\to-\infty} \tan^{-1} x$.

Example 2: Find the infinite limits, limits at infinity, and asymptotes for the function f whose graph is shown below.



Example 2: Find the following limits.

a) $\lim_{x\to\infty} \frac{1}{7x+1}$

b) $\lim_{x\to\infty} \sin x$

c) $\lim_{x\to\infty} 3e^{-x}$

How to Determine Limits at Infinity: Divide the numerator and denominator by the highest common power between the numerator and denominator.

Example 3: Find the limit.

(a) $\lim_{x\to\infty} \frac{2x+5}{x-4}$

(b) $\lim_{x\to\infty} \frac{x+4}{x^2+x-3}$

Example 4: Evaluate the following limits.

(a) $\lim_{x\to\infty} \frac{2x^2+5}{3x^2+1}$

(b) $\lim_{x\to\infty} \frac{2x+5}{3x^2+1}$

(c) $\lim_{x\to\infty} \frac{2x^3+5}{3x^2+1}$

Example 5: Find the following limits at infinity.

(a) $\lim_{x\to\infty} \dfrac{1+5e^x}{7-e^x}$

(b) $\lim_{x\to\infty} [\ln(2+x) - \ln(1+x)]$

Example 6: Find the limit.

(a) $\lim_{x\to\infty} \dfrac{x+2}{\sqrt{9x^2+1}}$

(b) $\lim_{x\to\infty} \dfrac{\sqrt{3x^6-x}}{x^3+1}$

How do deal with limits as $x \to -\infty$: Replace x by $-x$ and take the limit as $x \to \infty$.

Example 7: Find the limit.

(a) $\lim_{x\to-\infty} \dfrac{2x}{\sqrt{x^2+2}}$

(b) $\lim_{x\to-\infty} (5-3e^x)$

Example 8: Evaluate the following limits.

(a) $\lim_{x\to\infty}(\sqrt{x^4+6x^2}-x^2)$

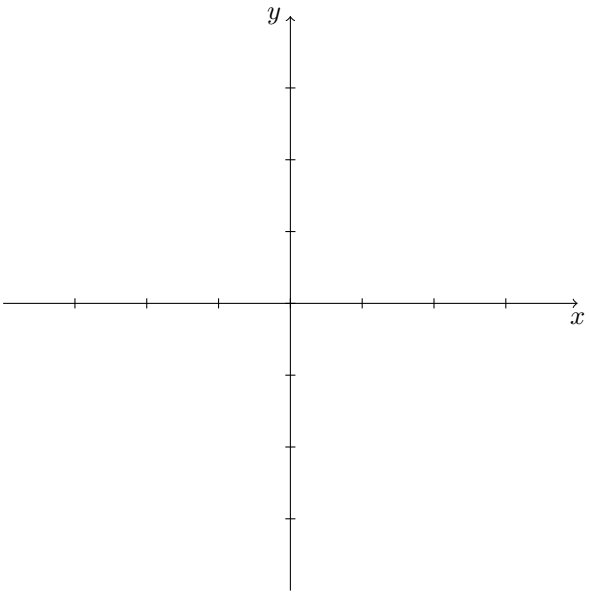
(b) $\lim_{x\to\infty}(\sqrt{x^2+1}-x)$

Example 9: Evaluate the following limits.

(a) $\lim_{x\to 0^-} e^{1/x}$

(b) $\lim_{x\to\infty} e^{-2x}\cos x$

Example 11: Sketch the graph of $y = (x-2)^4(x+1)^3(x-1)$ by finding its intercepts and its limits as $x \to \pm\infty$.



Example 12: Find the horizontal and vertical asymptotes of $f(x) = \dfrac{\sqrt{16x^2+1}}{2x-8}$.